ADVANCED CALCULUS

INTRODUCTORY REVIEW

CHAPTER I

REVIEW OF FUNDAMENTAL RULES

1. On differentiation. If the function $f(x)$ is interpreted as the curve $y = f(x)$,* the quotient of the increments Δy and Δx of the dependent and independent variables measured from (x_0, y_0) is

$$\frac{y - y_0}{x - x_0} = \frac{\Delta y}{\Delta x} = \frac{\Delta f(x)}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}, \tag{1}$$

and represents the *slope of the secant* through the points $P(x_0, y_0)$ and $P'(x_0 + \Delta x, y_0 + \Delta y)$ on the curve. The limit approached by the quotient $\Delta y / \Delta x$ when P remains fixed and $\Delta x \doteq 0$ is the *slope of the tangent* to the curve at the point P. *This limit,*

$$\lim_{\Delta x \doteq 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \doteq 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = f'(x_0), \tag{2}$$

is called the *derivative* of $f(x)$ for the value $x = x_0$. As the derivative may be computed for different points of the curve, it is customary to speak of the derivative as itself a function of x and write

$$\lim_{\Delta x \doteq 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \doteq 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = f'(x). \tag{3}$$

There are numerous notations for the derivative, for instance

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = D_x f = D_x y = y' = Df = Dy.$$

* Here and throughout the work, where figures are not given, the reader should draw graphs to illustrate the statements. Training in making one's own illustrations, whether graphical or analytic, is of great value.