IV. THE INVERSE SYSTEM AND MODULAR EQUATIONS

57. A considerable number of the properties proved in this section are to be found in (M); but the introduction of the inverse system is new.

Definitions. The array of the coefficients of a complete linearly independent set of members of a module M of degree $\leqslant t$ arranged under the power products $\omega_1, \omega_2, \ldots, \omega_\mu$ of degree $\leqslant t$ is called the *dialytic array* of the module M for degree t.

The linear homogeneous equations of which this array is the array of the coefficients are called the *dialytic equations* of M for degree t.

Thus the dialytic equations of M for degree t are represented by equating all members of M of degree $\leqslant t$ to zero and regarding the power products of $x_1, x_2, \ldots, x_n$ as symbols for the unknowns.

The array inverse (§ 54) to the dialytic array of M for degree t is called the *inverse array* of M for degree t.

The linear homogeneous equations of which this array is the array of the coefficients are called the *modular equations* of M for degree t.

The modular equations for degree t are the equations which are identically satisfied by the coefficients of each and every member of M of degree $\leqslant t$. They may not be independent for members of degree $< t$ and they do not apply to members of degree $> t$ (see § 59).

The sum of the products of the elements in any row of the inverse array for degree t with the inverse power products $\omega_1^{-1}, \omega_2^{-1}, \ldots, \omega_\mu^{-1}$ is called an *inverse function* of M for degree t.

Thus the modular equations of M for degree t are represented by equating all the inverse functions of M for degree t to zero, taking each negative power product $(x_1^{p_1} x_2^{p_2} \ldots x_n^{p_n})^{-1}$ as a symbol for "the coefficient of $x_1^{p_1} x_2^{p_2} \ldots x_n^{p_n}$ in the general member of M of degree t."

We shall also say that a polynomial $F = \Sigma a_{p_1, \ldots, p_n} x_1^{p_1} \ldots x_n^{p_n}$ and a finite or infinite negative power series $E = \Sigma c_{q_1, \ldots, q_n} (x_1^{q_1} \ldots x_n^{q_n})^{-1}$ are inverse to one another if the constant term of the product FE vanishes, i.e. if $\Sigma a_{p_1, p_2, \ldots, p_n} c_{p_1, p_2, \ldots, p_n} = 0$. Thus any member of M of degree $\leqslant t$ and any inverse function of M for degree t are inverse to one another.