I. THE RESULTANT

1. The Resultant is defined in the first instance with respect to n *homogeneous* polynomials $F_1, F_2, \ldots, F_n$ in n variables, of degrees $l_1, l_2, \ldots, l_n$, each polynomial being complete in all its terms with literal coefficients, all different. The resultant of any n given homogeneous polynomials in n variables is the value which the resultant in the general case assumes for the given case. The resultant of n given non-homogeneous polynomials in $n-1$ variables is the resultant of the corresponding homogeneous polynomials of the same degrees obtained by introducing a variable x_0 of homogeneity.

Definitions. An *elementary member* of the module $(F_1, F_2, \ldots, F_n)$ is any member of the type ωF_i $(i=1, 2, \ldots, n)$, where ω is any power product of $x_1, x_2, \ldots, x_n$. What is and what is not an elementary member depends on the basis chosen for the module.

The total number of elementary members of an assigned degree is evidently finite.

The diagram below represents the array of the coefficients of all elementary members of $(F_1, F_2, \ldots, F_n)$ of degree t, arranged under the power products $\omega_1^{(t)}, \omega_2^{(t)}, \ldots, \omega_\mu^{(t)}$ of degree t $\left(\mu = \dfrac{\lfloor t+n-1}{\lfloor t \ \lfloor n-1}\right)$:

$$\begin{array}{c|cccc|}
 & \omega_1^{(t)} & \omega_2^{(t)} & \ldots\ldots\ldots & \omega_\mu^{(t)} \\
\hline
\lambda_1 & a_1 & b_1 & \ldots\ldots\ldots & k_1 \\
\lambda_2 & a_2 & b_2 & \ldots\ldots\ldots & k_2 \\
 & \ldots & \ldots & \ldots\ldots\ldots & \ldots \\
\lambda_\rho & a_\rho & b_\rho & \ldots\ldots\ldots & k_\rho
\end{array}$$

Each row of the array, in association with $\omega_1^{(t)}, \omega_2^{(t)}, \ldots, \omega_\mu^{(t)}$, represents an elementary member of degree t; and the rows of the array corresponding to F_i all consist of the same elements (the coefficients of F_i and zeros) but in different columns.

Any member $F = X_1 F_1 + X_2 F_2 + \ldots + X_n F_n$ of degree t is evidently a linear combination $\lambda_1\omega_1 F_1 + \lambda_2\omega_2 F_1 + \ldots + \lambda_p\omega_p F_i + \ldots + \lambda_\rho\omega_\rho F_n$ of elementary members of degree t, and is represented by the above array when bordered by $\lambda_1, \lambda_2, \ldots, \lambda_\rho$ on the left, where $\lambda_1, \lambda_2, \ldots, \lambda_\rho$ are the coefficients of $X_1, X_2, \ldots, X_n$, some of which may be zeros.

This bordered array also shows in a convenient way the whole coefficients of the terms of F, viz. $\Sigma\lambda a, \Sigma\lambda b, \ldots, \Sigma\lambda k$.