THE ALGEBRAIC THEORY OF MODULAR SYSTEMS

Introduction

Definition. A modular system is an infinite aggregate of polynomials, or whole functions* of n variables $x_1, x_2, \ldots, x_n$, defined by the property that if F, F_1, F_2 belong to the system $F_1 + F_2$ and AF also belong to the system, where A is any polynomial in $x_1, x_2, \ldots, x_n$.

Hence if $F_1, F_2, \ldots, F_k$ belong to a modular system so also does $A_1F_1 + A_2F_2 + \ldots + A_kF_k$, where $A_1, A_2, \ldots, A_k$ are arbitrary polynomials.

Besides the algebraic or relative theory of modular systems there is a still more difficult and varied absolute theory. We shall only consider the latter theory in so far as it is necessary for the former.

In the algebraic theory polynomials such as F and aF, where a is a quantity not involving the variables, are not regarded as different polynomials, and any polynomial of degree zero is equivalent to 1. No restriction is placed on the coefficients of $F_1, F_2, \ldots, F_k$ except in so far as they may involve arbitrary parameters $u_1, u_2, \ldots$, in which case they are restricted to being rational functions of such parameters. The same restriction applies to the coefficients of the arbitrary polynomials $A_1, A_2, \ldots, A_k$ above.

In the absolute theory the coefficients of $F_1, F_2, \ldots, A_1, A_2, \ldots$ are restricted to a domain of integrity, generally ordinary integers or whole functions of parameters $u_1, u_2, \ldots$ with integral coefficients; and a polynomial of degree zero other than 1 or a unit is not equivalent to 1.

* We use the term *whole function* throughout the text (but not in the Note at the end) as equivalent to *polynomial* and as meaning a *whole rational function.*