PART III

SYMBOLIC NOTATION

THE NOTATION AND ITS IMMEDIATE CONSEQUENCES, §§ 39–41

39. Introduction. The conditions that the binary cubic

(1) $$f = a_0x_1^3 + 3a_1x_1^2x_2 + 3a_2x_1x_2^2 + a_3x_2^3$$

shall be a perfect cube

(2) $$(\alpha_1x_1 + \alpha_2x_2)^3$$

are found by eliminating α_1 and α_2 between

(3) $$\alpha_1^3 = a_0, \qquad \alpha_1^2\alpha_2 = a_1, \qquad \alpha_1\alpha_2^2 = a_2, \qquad \alpha_2^3 = a_3,$$

and hence the conditions are

(4) $$a_0a_2 = a_1^2, \qquad a_1a_3 = a_2^2.$$

Thus only a very special form (1) is a perfect cube.

However, in a symbolic sense * any form (1) can be represented as a cube (2), in which α_1 and α_2 are now mere symbols such that

(3′) $$\alpha_1^3, \qquad \alpha_1^2\alpha_2, \qquad \alpha_1\alpha_2^2, \qquad \alpha_2^3$$

are given the interpretations (3), while any linear combination of these products, as $2\alpha_1^3 - 7\alpha_2^3$, is interpreted to be the corresponding combination of the a's, as $2a_0 - 7a_3$. But no interpretation is given to a polynomial in α_1, α_2, any one of whose terms is a product of more than three factors α, or fewer than three factors α. Thus the first relation (4) does not now follow from (3), since the expression $\alpha_1^4\alpha_2^2$ (formerly equal to both

* Due to Aronhold and Clebsch, but equivalent to the more complicated hyperdeterminants of Cayley.