PART II

THEORY OF INVARIANTS IN NON-SYMBOLIC NOTATION

15. Homogeneity of Invariants. We saw in § 11 that two binary quadratic forms f and f' have the invariants

$$d = ac - b^2, \qquad s = ac' + a'c - 2bb'$$

of index 2. Note that s is of the first degree in the coefficients a, b, c of f and also of the first degree in the coefficients of f', and hence is homogeneous in the coefficients of each form separately. The latter is also true of d, but not of the invariant $s+2d$.

When an invariant of two or more forms is not homogeneous in the coefficients of each form separately, it is a sum of invariants each homogeneous in the coefficients of each form separately.

A proof may be made similar to that used in the following case. Grant merely that $s+2d$ is an invariant of index 2 of the binary quadratic forms f and f'. In the transformed forms (§ 11), the coefficients A, B, C of F are linear in a, b, c; the coefficients A', B', C' of F' are linear in a', b', c'. By hypothesis

$$AC' + A'C - 2BB' + 2(AC - B^2) = \Delta^2(s + 2d).$$

The terms $2d\Delta^2$ of degree 2 in a, b, c on the right arise only from the part $2(AC - B^2)$ on the left. Hence d is itself an invariant of index 2; likewise s itself is an invariant.

However, *an invariant of a single form is always homogeneous.* For example, this is the case with the above discriminant d of f. We shall deduce this theorem from a more general one.