In formulas (1) and (3), in which b is any term at all, we might introduce the sign $\prod$ with respect to b. In the following formula, it becomes necessary to make use of this sign.

4. $$\prod_x \left\{[a < (b < x)] < x\right\} = ab.$$

Demonstration:

$$\left\{[a < (b < x)] < x\right\} = \left\{[a' + (b < x)] < x\right\}$$

$$= [(a' + b' + x) < x] = abx' + x = ab + x.$$

We must now form the product $\prod_x (ab + x)$, where x can assume every value, including 0 and 1. Now, it is clear that the part common to all the terms of the form $(ab + x)$ can only be ab. For, (1) ab is contained in each of the sums $(ab + x)$ and therefore in the part common to all; (2) the part common to all the sums $(ab + x)$ must be contained in $(ab + 0)$, that is, in ab. Hence this common part is equal to ab[1], which proves the theorem.

59. Reduction of Inequalities to Equalities.—As we have said, the principle of assertion enables us to reduce inequalities to equalities by means of the following formulas:

$$(a \neq 0) = (a = 1), \qquad (a \neq 1) = (a = 0),$$

$$(a \neq b) = (a = b').$$

For,

$$(a \neq b) = (ab' + a'b \neq 0) = (ab' + ab' = 1) = (a = b').$$

Consequently, we have the paradoxical formula

$$(a \neq b) = (a = b').$$

1 This argument is general and from it we can deduce the formula

$$\prod_x (a + x) = a,$$

whence may be derived the correlative formula

$$\sum_x ax = a.$$