Finally, in the third, the product of two propositions cannot be false unless one of them is false, for, if both were true, their product would be true (equal to 1).

58. Law of Importation and Exportation.—The fundamental equivalence $(a < b) = a' + b$ has many other interesting consequences. One of the most important of these is *the law of importation and exportation,* which is expressed by the following formula:

$$[a < (b < c)] = (ab < c).$$

"To say that if a is true b implies c, is to say that a and b imply c".

This equality involves two converse implications: If we infer the second member from the first, we *import* into the implication $(b < c)$ the hypothesis or condition a; if we infer the first member from the second, we, on the contrary, *export* from the implication $(ab < c)$ the hypothesis a.

Demonstration:

$$[a < (b < c)] = a' + (b < c) = a' + b' + c,$$

$$(ab < c) = (ab)' + c = a' + b' + c.$$

Cor. 1.—Obviously we have the equivalence

$$[a < (b < c)] = [b < (a < c)],$$

since both members are equal to $(ab < c)$, by the commutative law of multiplication.

Cor. 2.—We have also

$$[a < (a < b)] = (a < b),$$

for, by the law of importation and exportation,

$$[a < (a < b)] = (aa < b) = (a < b).$$

If we apply the law of importation to the two following formulas, of which the first results from the principle of identity and the second expresses the principle of contraposition,