

But now, according to Ax. X, we have

$$(a = 1) + (a = 0) = a + a' = 1.$$

“A proposition is either true or false”. From these two formulas combined we deduce directly that the propositions $(a = 1)$ and $(a = 0)$ are contradictory, *i. e.*,

$$(a \neq 1) = (a = 0), \quad (a \neq 0) = (a = 1).$$

From the point of view of calculation Ax. X makes it possible to reduce to its first member every equality whose second member is 1, and to transform inequalities into equalities. Of course these equalities and inequalities must have propositions as their members. Nevertheless all the formulas of this section are also valid for classes in the particular case where the universe of discourse contains only one element, for then there are no classes but 0 and 1. In short, the special calculus of propositions is equivalent to the calculus of classes when the classes can possess only the two values 0 and 1.

57. Equivalence of an Implication and an Alternative.

—The fundamental equivalence

$$(a < b) = (a' + b = 1)$$

gives rise, by Ax. X, to the equivalence

$$(a < b) = (a' + b),$$

which is no less fundamental in the calculus of propositions. To say that a implies b is the same as affirming “not- a or b ”, *i. e.*, “either a is false or b is true.” This equivalence is often employed in every day conversation.

Corollary.—For any equality, we have the equivalence

$$(a = b) = ab + a'b'.$$

Demonstration:

$$(a = b) = (a < b) (b < a) = (a' + b) (b' + a) = ab + a'b'.$$

“To affirm that two propositions are equal (equivalent) is the same as stating that either both are true or both are false”.