

45. **The Law of Causes.**—The method of finding the consequences of a given equality suggests directly the method of finding its *causes*, namely, the propositions of which it is the consequence. Since we pass from the cause to the consequence by eliminating known terms, *i. e.*, by suppressing constituents, we will pass conversely from the consequence to the cause by adjoining known terms, *i. e.*, by adding constituents to the given equality. Now, the number of constituents that may be added to it, *i. e.*, that do not already appear in it, is $2^n - m$. We will obtain all the possible causes (in the universe of the n terms under consideration) by forming all the additive combinations of these constituents, and adding them to the first member of the equality in virtue of the general formula

$$(A + B = 0) < (A = 0),$$

which means that the equality $(A = 0)$ has as its cause the equality $(A + B = 0)$, in which B is any term. The number of causes thus obtained will be equal to the number of the aforesaid combinations, or $2^{2^n - m}$.

This method may be applied to the investigation of the causes of the premises of the syllogism

$$(a < b) (b < c),$$

which, as we have seen, is equivalent to the developed equality

$$abc' + ab'c + ab'c' + a'bc' = 0.$$

This equality contains four of the eight (2^3) constituents of the universe of three terms, the four others being

$$abc, a'bc, a'b'c, a'b'c'.$$

The number of their combinations is 16 (2^4), this is also the number of the causes sought, which are:

1. $(abc + abc' + ab'c + ab'c' + a'bc' = 0)$
 $= (a + bc' = 0) = (a = 0) (b < c);$
2. $(ab'c + ab'c + ab'c' + a'bc + a'bc' = 0)$
 $= (ab'c' + ab' + a'b = 0) = (ab < c) (a = b);$