

21. **The Developments of 0 and of 1.**—Hitherto we have met only such formulas as directly express customary modes of reasoning and consequently offer direct evidence.

We shall now expound theories and methods which depart from the usual modes of thought and which constitute more particularly the algebra of logic in so far as it is a formal and, so to speak, automatic method of an absolute universality and an infallible certainty, replacing reasoning by calculation.

The fundamental process of this method is *development*. Given the terms  $a, b, c \dots$  (to any finite number), we can develop 0 or 1 with respect to these terms (and their negatives) by the following formulas derived from the distributive law:

$$0 = aa',$$

$$0 = aa' + bb' = (a + b)(a + b')(a' + b)(a' + b'),$$

$$0 = aa' + bb' + cc' = (a + b + c)(a + b + c')(a + b' + c)$$

$$\times (a + b' + c')(a' + b + c)$$

$$\times (a' + b + c')(a' + b' + c)(a' + b' + c');$$

$$1 = a + a',$$

$$1 = (a + a')(b + b') = ab + ab' + a'b + a'b',$$

$$1 = (a + a')(b + b')(c + c') = abc + abc' + ab'c + ab'c'$$

$$+ a'bc + a'bc' + a'b'c + a'b'c';$$

and so on. In general, for any number  $n$  of simple terms, 0 will be developed in a product containing  $2^n$  factors, and 1 in a sum containing  $2^n$  summands. The factors of zero comprise all possible additive combinations, and the summands of 1 all possible multiplicative combinations of the  $n$  given terms and their negatives, each combination comprising  $n$  different terms and never containing a term and its negative at the same time.

The summands of the development of 1 are what BOOLE called the *constituents* (of the universe of discourse). We may equally well call them, with PORETSKY,<sup>1</sup> the *minima* of discourse, because they are the smallest classes into which the

<sup>1</sup> See the Bibliography, page xiv.