On the other hand: 1. Add a' to each of the two members of the inclusion $a < b$; we have

$$(a' + a < a' + b) = (1 < a' + b) = (a' + b = 1).$$

2. We know that

$$b = (a + b)\ (a' + b).$$

Now, if $a' + b = 1$,

$$b = (a + b) \times 1 = a + b.$$

By the preceding formulas, an inclusion can be transformed at will into an equality whose second member is either 0 or 1. Any equality may also be transformed into an equality of this form by means of the following formulas:

$$(a = b) = (ab' + a'b = 0), \quad (a = b) = [(a + b')\ (a' + b) = 1].$$

Demonstration:

$$(a = b) = (a < b)\ (b < a) = (ab' = 0)\ (a'b = 0) = (ab' + a'b = 0),$$

$$(a = b) = (a < b)\ (b < a) = (a' + b = 1)\ (a + b' = 1) =$$
$$[(a' + b')\ (a' + b) = 1].$$

Again, we have the two formulas

$$(a = b) = [(a + b)\ (a' + b') = 0], \quad (a = b) = (ab + a'b' = 1),$$

which can be deduced from the preceding formulas by performing the indicated multiplications (or the indicated additions) by means of the distributive law.

19. Law of Contraposition.—We are now able to demonstrate the *law of contraposition*,

$$(a < b) = (b' < a').$$

Demonstration.—By the preceding formulas, we have

$$(a < b) = (ab' = 0) = (b' < a').$$

Again, the law of contraposition may take the form

$$(a < b') = (b < a'),$$

which presupposes the law of double negation. It may be expressed verbally as follows: "Two members of an inclusion may be interchanged on condition that both are denied".