

This law may be expressed in the following manner:

If $b = a'$, we have $a = b'$, and conversely, by symmetry.

This proposition makes it possible, in calculations, to transpose the negative from one member of an equality to the other.

The law of double negation makes it possible to conclude the equality of two terms from the equality of their negatives (if $a' = b'$ then $a = b$), and therefore to cancel the negation of both members of an equality.

From the characteristic formulas of negation together with the fundamental properties of 0 and 1, it results that every product which contains two contradictory factors is null, and that every sum which contains two contradictory summands is equal to 1.

In particular, we have the following formulas:

$$a = ab + ab', \quad a = (a + b) (a + b'),$$

which may be demonstrated as follows by means of the distributive law:

$$\begin{aligned} a &= a \times 1 = a(b + b') = ab + ab', \\ a &= a + 0 = a + bb' = (a + b) (a + b'). \end{aligned}$$

These formulas indicate the principle of the method of development which we shall explain in detail later (§§ 21 sqq.)

18. Second Formula for Transforming Inclusions into Equalities:—We can now establish two very important equivalences between inclusions and equalities:

$$(a < b) = (ab' = 0), \quad (a < b) = (a' + b = 1).$$

Demonstration.—1. If we multiply the two members of the inclusion $a < b$ by b' we have

$$(ab' < bb') = (ab' < 0) = (ab' = 0).$$

2. Again, we know that

$$a = ab + ab'.$$

Now if $ab' = 0$,

$$a = ab + 0 = ab.$$