

P. I.: 1. The simultaneous affirmation of the propositions a and not- a is false; in other words, these two propositions cannot both be true at the same time.

2. The alternative affirmation of the propositions a and not- a is true; in other words, one of these two propositions must be true.

Two propositions are said to be *contradictory* when one is the negative of the other; they cannot both be true or false at the same time. If one is true the other is false; if one is false the other is true.

This is in agreement with the fact that the terms 0 and 1 are the negatives of each other; thus we have

$$0 \times 1 = 0, \quad 0 + 1 = 1.$$

Generally speaking, we say that two terms are *contradictory* when one is the negative of the other.

17. Law of Double Negation.—Moreover this reciprocity is general: if a term b is the negative of the term a , then the term a is the negative of the term b . These two statements are expressed by the same formulas

$$ab = 0, \quad a + b = 1,$$

and, while they unequivocally determine b in terms of a , they likewise determine a in terms of b . This is due to the symmetry of these relations, that is to say, to the commutativity of multiplication and addition. This reciprocity is expressed by the *law of double negation*

$$(a')' = a,$$

which may be formally proved as follows: a' being by hypothesis the negative of a , we have

$$aa' = 0, \quad a + a' = 1.$$

On the other hand, let a'' be the negative of a' ; we have, in the same way,

$$a'a'' = 0, \quad a' + a'' = 1.$$

But, by the preceding lemma, these four equalities involve the equality

$$a = a''.$$

Q. E. D.