the definition. We will prove that they are equal. Since, by hypothesis,

$$aa'_1 = 0, \quad a + a'_1 = 1,$$
$$aa'_2 = 0, \quad a + a'_2 = 1,$$

we have

$$aa'_1 = aa'_2, \quad a + a'_1 = a + a'_2;$$

whence we conclude, by the preceding lemma, that

$$a'_1 = a'_2.$$

We can now speak of *the* negative of a term as of a unique and well-defined term.

The *uniformity* of the operation of negation may be expressed in the following manner:

If $a = b$, then also $a' = b'$. By this proposition, both members of an equality in the logical calculus may be "denied".

16. The Principles of Contradiction and of Excluded Middle.—By definition, a term and its negative verify the two formulas

$$aa' = 0, \quad a + a' = 1,$$

which represent respectively the *principle of contradiction* and the *principle of excluded middle.*[1]

C. I.: 1. The classes a and a' have nothing in common; in other words, no element can be at the same time both a and not-a.

2. The classes a and a' combined form the whole; in other words, every element is either a or not-a.

---

[1] As Mrs. LADD-FRANKLIN has truly remarked (BALDWIN, *Dictionary of Philosophy and Psychology*, article "Laws of Thought"), the principle of *contradiction* is not sufficient to define *contradictories;* the principle of excluded middle must be added which equally deserves the name of principle of contradiction. This is why Mrs. LADD-FRANKLIN proposes to call them respectively the *principle of exclusion* and the *principle of exhaustion,* inasmuch as, according to the first, two contradictory terms are *exclusive* (the one of the other); and, according to the second, they are *exhaustive* (of the universe of discourse).