classes is the part that is common to each (the class of their common elements) and the sum of two classes is the class of all the elements which belong to at least one of them.

P. I.: 1. The product of two propositions is a proposition which implies each of them and which is implied by every proposition which implies both:

2. The sum of two propositions is the proposition which is implied by each of them and which implies every proposition implied by both.

Therefore we can say that the product of two propositions is their weakest common cause, and that their sum is their strongest common consequence, strong and weak being used in a sense that every proposition which implies another is stronger than the latter and the latter is weaker than the one which implies it. Thus it is easily seen that the product of two propositions consists in their *simultaneous affirmation:* "a and b are true", or simply "a and b"; and that their sum consists in their *alternative affirmation*, "either a or b is true", or simply "a or b".

Remark.—Logical addition thus defined is not disjunctive;[1] that is to say, it does not presuppose that the two summands have no element in common.

8. Principles of Simplification and Composition.—The two preceding definitions, or rather the postulates which precede and justify them, yield directly the following formulas:

(1) $$ab < a, \qquad ab < b,$$

(2) $$(x < a)\,(x < b) < (x < ab),$$

(3) $$a < a + b, \qquad b < a + \mathrm{b},$$

(4) $$(a < x)\,(b < x) < (a + b < x).$$

Formulas (1) and (3) bear the name of the *principle of simplification* because by means of them the premises of an

1 [BOOLE, closely following analogy with ordinary mathematics, premised, as a necessary condition to the definition of "$x + y$", that x and y were mutually exclusive. JEVONS, and practically all mathematical logicians after him, advocated, on various grounds, the definition of "logical addition" in a form which does not necessitate mutual exclusiveness.]