

We see that in this formula the principal copula has always the sense of implication because the proposition is a secondary one.

By the definition of equality the consequences of the principle of the syllogism may be stated in the following formulas¹:

$$\begin{aligned}(a < b) \quad (b = c) &< (a < c), \\(a = b) \quad (b < c) &< (a < c), \\(a = b) \quad (b = c) &< (a = c).\end{aligned}$$

The conclusion is an equality only when both premises are equalities.

The preceding formulas can be generalized as follows:

$$\begin{aligned}(a < b) \quad (b < c) \quad (c < d) &< (a < d), \\(a = b) \quad (b = c) \quad (c = d) &< (a = d).\end{aligned}$$

Here we have the two chief formulas of the *sorites*. Many other combinations may be easily imagined, but we can have an equality for a conclusion only when all the premises are equalities. This statement is of great practical value. In a succession of deductions we must pay close attention to see if the transition from one proposition to the other takes place by means of an equivalence or only of an implication. There is no equivalence between two extreme propositions unless all intermediate deductions are equivalences; in other words, if there is one single implication in the chain, the relation of the two extreme propositions is only that of implication.

7. Multiplication and Addition.—The algebra of logic admits of three operations, *logical multiplication*, *logical addition*, and *negation*. The two former are binary operations, that is to say, combinations of two terms having as a consequent a third term which may or may not be different from each of them. The existence of the logical product and logical sum of two terms must necessarily answer the purpose of a

¹ Strictly speaking, these formulas presuppose the laws of multiplication which will be established further on; but it is fitting to cite them here in order to compare them with the principle of the syllogism from which they are derived.