CHAPTER 14

Further problems

14.1. Multiplicities of spectral parameters

Suppose a Fuchsian differential equation and its middle convolution are given. Then we can analyze the corresponding transformation of a global structure of its local solution associated with an eigenvalue of the monodromy generator at a singular point if the eigenvalue is free of multiplicity.

When the multiplicity of the eigenvalue is larger than one, we have not a satisfactory result for the transformation (cf. Theorem 12.5). The value of a generalized connection coefficient defined by Definition 12.17 may be interesting. Is the procedure in Remark 12.19 always valid? In particular, is there a general result assuring Remark 12.19 (1) (cf. Remark 12.23)? Are the multiplicities of zeros of the generalized connection coefficients of a rigid Fuchsian differential equation free?

14.2. Schlesinger canonical form

Can we define a natural *universal* Fuchsian system of Schlesinger canonical form (1.79) with a given realizable spectral type? Here we recall Example 9.2.

Let $P_{\mathbf{m}}$ be the universal operator in Theorem 6.14. Is there a natural system of Schlesinger canonical form which is isomorphic to the equation $P_{\mathbf{m}}u = 0$ together with the explicit correspondence between them?

14.3. Apparent singularities

Katz [**Kz**] proved that any irreducible rigid local system is constructed from the trivial system by successive applications of middle convolutions and additions and it is proved in this paper that the system is realized by a single differential equation without an apparent singularity.

In general, an irreducible local system cannot be realized by a single differential equation without an apparent singularity but it is realized by that with apparent singularities. Hence it is expected that there exist some natural operations of single differential equations with apparent singularities which correspond to middle convolutions of local systems or systems of Schlesinger canonical form.

The Fuchsian ordinary differential equation satisfied by an important special function often hasn't an apparent singularity even if the spectral type of the equation is not rigid. Can we understand the condition that a $W(x)$-module has a generator so that it satisfies a differential equation without an apparent singularity? Moreover it may be interesting to study the existing of contiguous relations among differential equations with fundamental spectral types which have no apparent singularity.

14.4. Irregular singularities

Our fractional operations defined in Chapter 1 give transformations of ordinary differential operators with polynomial coefficients, which have irregular singularities in general. The reduction of ordinary differential equations under these operations