Chapter 8

Deformation spaces of real projective structures on 2-orbifolds of negative Euler characteristic: An introduction

The main purpose here is to introduce real projective structures on 2-orbifolds to the readers. The theoretical aspects are not completely written here but the readers can find them in articles mentioned. Additionally, we discuss the computational aspect of this theory in a more detailed way.

First, we will give some introduction to real projective structures on orbifolds with relationships to hyperbolic structures. Next, we give some examples of real projective structures on annuli, a torus with one-hole and the orbifolds based on a triangle.

We also give a survey of real projective structures on manifolds (and orbifolds) from a historical point of view: the Hilbert metrics, the topological work of Choi (1994a,b) and Goldman (1990), the gauge theory point of view using Higgs bundles, the Hitchin's conjecture and the group theoretical work of Benoist (2001).

Next, we study real projective structures on 2-orbifolds of negative Euler characteristic. We present Theorem 8.3.1 characterizing the topology of the deformation spaces of convex real projective structures on 2-orbifolds of negative Euler characteristic. Next, we study the relationship between the deformation spaces and the Hitchin-Teichmüller components of the spaces of $\mathbb{PGL}(3, \mathbb{R})$-characters in Section 8.3.1. We try to now understand the deformation spaces of real projective structures on orbifolds. We discuss the geometric constructions available for such structures and the elementary 2-orbifolds and their real projective structures using the work of Goldman (1990). From these, we should be able to prove Theorem 8.3.1 characterizing the topology of the deformation space of real projective structures on 2-orbifolds. However, we do not present the full detail.

8.1 Introduction to real projective orbifolds

Let X be the real projective plane $\mathbb{RP}^n$ and G the group $\mathbb{PGL}(n+1, \mathbb{R})$ of *collineations*, i.e., projective automorphisms of $\mathbb{RP}^n$. An $\mathbb{RP}^n$*-structure* or *real projective structure* on an n-dimensional orbifold Σ is an $(\mathbb{RP}^n, \mathbb{PGL}(n+1, \mathbb{R}))$-structure on Σ. Two $\mathbb{RP}^n$-structures on Σ are equivalent if an isotopy from the identity map