Chapter 6

Geometry of orbifolds: geometric structures on orbifolds

In this section, we introduce the geometric structures on orbifolds. The definition is given by the method of atlases of charts, making use of (G, X)-pseudo group structures in Section 2.3. We show that geometric orbifolds are always good by using the foliation theory, an important result due to Thurston (See Chapter 5 of the book [Thurston (1977)].) Then we discuss developing maps, global charts, and associated holonomy homomorphisms. These can also be used as definitions of geometric structures. We also introduce the approach using flat bundles and transverse sections to define the geometric structures. (See Section 2.4.) These observations were first due to Goldman (1987) for manifolds. The article [Goldman (2010)] contains a general introduction to geometric structures on manifolds.

Next, we introduce the deformation spaces of geometric structures on orbifolds using the above three approaches as were done by Goldman for manifolds. We finally mention the local homeomorphism theorem from the deformation space to the representation space.

6.1 The definition of geometric structures on orbifolds

Let (G, X) be a pair defining a geometry. That is, G is a Lie group acting on a manifold effectively and transitively. Let M be a connected n-orbifold with boundary, possibly empty. We have three ways to define a (G, X)-geometric structure on M:

- Atlases of charts.
- A developing map from the universal covering space.
- A cross-section of the flat orbifold X-bundle.

6.1.1 *An atlas of charts approach*

Given an imbedding $f : U \to V$ between two domains U and V in $\mathbb{R}^n$ with groups G_1 and G_2 acting on them respectively, we denote by $f^* : G_1 \to G_2$ the homomorphism determined by sending $\vartheta \in G_1$ to the element of G_2 agreeing with $f \circ \vartheta \circ f^{-1}$ in an