Chapter 2

Manifolds and $\mathcal{G}$-structures

In this chapter, we review many notions in the manifold theory that can be generalized to the orbifold theory.

We begin by reviewing manifolds and simplicial manifolds beginning with cell-complexes and the homotopy and covering theory. The following theories for manifolds will be transferred to orbifolds. We briefly mention them here as a "review" and develop them for orbifolds later (mostly for 2-dimensional orbifolds):

- Lie groups and group actions
- Pseudo-groups and $\mathcal{G}$-structures
- Differential geometry: Riemanian manifolds, principal bundles, connections, and flat connections

We follow a coordinate-free approach to differential geometry. We do not need to actually compute curvatures and so on.

Some of these are standard materials in a differentiable manifold course. We will not give proofs in Chapters 2 and 3 but will indicate one when necessary.

2.1 The review of topology

We present a review of the manifold topology. We will find that many of these directly can be generalized into the orbifold theory later.

2.1.1 *Manifolds*

The useful methods of topology come from taking equivalence classes and finding quotient topology. Given a topological space X with an equivalence relation, we give the quotient topology on $X/\sim$ so that for any function $f : X \to Y$ inducing a well-defined function $f' : X/\sim \to Y$, f' is continuous if and only if f is continuous. This translates to the fact that a subset U of $X/\sim$ is open if and only if $p^{-1}(U)$ is open in X for the quotient map $p : X \to X/\sim$.

A *cell* is a topological space homeomorphic to an n-dimensional open convex