Chapter 1

Introduction

One aim of mathematics is to explore many objects purely defined and created out of imaginations in the hope that they would explain many unknown and unsolved phenomena in mathematics and other fields. As one knows, the manifold theory enjoyed a great deal of attention in the 20th century mathematics involving many talented mathematicians. Perhaps, mathematicians should develop more abstract theories that can accommodate many things that we promised to unravel in the earlier part of 20th century. The theory of orbifolds might be a small step in the right direction as orbifolds have all the notions of the manifold theory easily generalized as discovered by Satake and developed by Thurston. In fact, orbifolds have most notions developed from the manifold theory carried over to them although perhaps in an indirect manner, using the language of the category theory. Indeed, to make the orbifold theory most rigorously understood, only the category theory provides the natural settings.

Orbifolds provide a natural setting to study discrete group actions on manifolds, and orbifolds can be more useful than manifolds in many ways involving in the classification of knots, the graph embeddings, theoretical physics and so on. At least in two- or three-dimensions, orbifolds are much easier to produce and classifiable using Thurston's geometrization program. (See for example the program "Orb" by Heard and Hodgson (2007).) In particular, one obtains many examples with ease to experiment with. The subject of higher dimensional orbifolds is still very mysterious where many mathematicians and theoretical physicists are working on. In fact, the common notion that orbifolds are almost always covered by manifolds is not entirely relevant particularly for the higer-dimensional orbifolds. For example, these kinds of orbifolds might exist in abundance and might prove to be very useful. It is thought that orbifolds are integral part of theoretical physics such as the string theory, and they have natural generalizations in algebraic geometry as stacks.

For 2-manifolds, it was known from the classical times that the geometry provides a sharp insight into the topology of surfaces and their groups of automorphisms as observed by Dehn and others. In late 1970s, Thurston proposed a program to generalize these kinds of insights to the 3-manifold theory. This program is now completed by Perelman's proof of the Geometrization conjecture as is well-known.