CHAPTER 3

Non-Triviality of the Godbillon–Vey Class

The aim of this chapter is to show the following Theorem A in Introduction.

THEOREM A.

1) *For each q, there are transversely holomorphic foliations of complex codimension q of which the Godbillon–Vey classes are non-trivial.*
2) *If q is odd and $q \geq 3$, then there are at least two transversely holomorphic foliations of complex codimension q which are non-cobordant as real foliations of codimension $2q$. If $q = 5$, then there are at least three transversely holomorphic foliations such that none of them are cobordant as real foliations of real codimension 10.*

Moreover, these foliations can be realized as locally homogeneous foliations.

For this purpose, we will first introduce locally homogeneous foliations and then explain how their complex secondary classes are computed. We will show Theorem A in Section 3.3 by constructing examples. Similar examples in the real category are studied by several authors. See for example Baker [12] and the references therein.

3.1. Locally Homogeneous Foliations and Complex Secondary Classes

NOTATION 3.1.1. Given a Lie group, we denote its Lie algebra by the corresponding German lower case letter, e.g., if G is a Lie group, then its Lie algebra is denoted by $\mathfrak{g}$.

Let G be a Lie group and K its connected closed Lie subgroup. Let H be a connected subgroup of G which contains K, and denote by $\widetilde{\mathcal{F}}$ the foliation of G whose