Chapter 7

Appendices

§A1. A counter example

Consider the following Cauchy problem

$$\begin{cases} r_t + (1 + rs)r_x = 0, \\ s_t = 0, \end{cases} \tag{A1.1}$$

$$t = 0: \quad r = \varepsilon r_0(x),\ s = \varepsilon s_0(x), \tag{A1.2}$$

where $r_0(x)$ and $s_0(x)$ are C^1 functions with bounded C^1 norm, $\varepsilon > 0$ is a small parameter.

Obviously, in a neighbourhood of $(r, x) = (0, 0)$, (A1.1) is a strictly hyperbolic system with two distinct real eigenvalues

$$\lambda_1(r, s) \triangleq 1 + rs > \lambda_2(r, s) \triangleq 0. \tag{A1.3}$$

On the other hand, by Definition 3.1 it is easy to check that system (A1.1) is weakly linearly degenerate. Therefore, by Theorem 3.1 we have

Theorem A1.1. Under the hypotheses mentioned above, if $r_0(x)$ and $s_0(x)$ satisfy