

Proof-Theoretic Methods in Nonclassical Logic — an Introduction

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1 Introduction

This is an introduction to proof theory of nonclassical logic, which is directed at people who have just started the study of nonclassical logics, using proof-theoretic methods. In our paper, we will discuss only its proof theory based on sequent calculi. So, we will discuss mainly cut elimination and its consequences. As this is not an introduction to sequent systems themselves, we will assume a certain familiarity with standard sequent systems **LK** for the classical logic and **LJ** for the intuitionistic logic. When necessary, readers may consult e.g. Chapter 1 of *Proof Theory* [43] by Takeuti, Chapters 3 and 4 of *Basic Proof Theory* [45] by Troelstra and Schwichtenberg, and Chapter 1 of the present Memoir by M. Takahashi [41] to supplement our paper. Also, our intention is not to give an introduction of nonclassical logic, but to show how the standard proof-theoretic methods will work well in the study of nonclassical logic, and why certain modifications will be necessary in some cases. We will take only some modal logics and substructural logics as examples, and will give remarks on further applications. Notes at the end of each section include some indications for further reading.

An alternative approach to proof theory of nonclassical logic is given by using natural deduction systems. As it is well-known, natural deduction systems are closely related to sequent calculi. For instance, the normal form theorem in natural deduction systems corresponds to the cut elimination theorem in sequent calculi. So, it will be interesting to look for results and techniques on natural deduction systems which are counterparts of those on sequent calculi, given in the present paper.

The present paper is based on the talk given at *MSJ Regional Workshop on Theories of Types and Proofs* at Tokyo Institute of Technology in September, 1997, and also on the introductory talk on proof theory in Logic Summer School 1998, at Australian National University in January, 1998. The author would like to express