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Discrete topological methods for subspace arrangements

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§1. Introduction

A different (and relatively new) method to deal with the topology of Hyperplane Arrangements (and more generally Subspace Arrangements) is given by a discrete version of Morse Theory, namely the so called Discrete Morse Theory. Practically, this theory originated from the standard theory as a suitable adjustment to spaces usually produced by discrete data, like simplicial complexes and more generally CW-complexes (see [Fo98, Fo02, Ko08]).

In [SaSe07] we considered complement to *real* hyperplane arrangements applying Discrete Morse Theory to a well-known CW-complex with the same homotopy type ([Sa87]). We re-proved the *minimality* of the complement: the complement to a hyperplane arrangement is a *minimal space,* i.e. it has the homotopy type of a CW-complex with as many i-cells as its ith-Betti number ($i \geq 0$). This interesting result was proven independently in [DP03, Ra02] as an existence-like theorem; the explicit structure of the minimal complex was considered before us by [Yo05] and after us by [De08] (see also [DeSe]) .

The construction which uses Discrete Morse Theory is much more precise, even if superabundant in the description of the attaching maps of the cells (new "reduced" descriptions, at least in case of dimension two, were recently found by Yoshinaga himself and, by different method, by the two authors togheter with G. Gaiffi [GMS10]). This combinatorial method allows to produce algebraic complexes which calculate local

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