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Proportionality of indices of 1-forms on singular varieties

Jean-Paul Brasselet, José Seade* and Tatsuo Suwa†

§0. Introduction

M.-H. Schwartz in [20, 21] introduced the technique of radial extension of stratified vector fields and frames on singular varieties, and used this to construct cocycles representing classes in the cohomology $H^*(M, M \setminus V)$, where V is a singular variety embedded in a complex manifold M; these are now called *the Schwartz classes* of V. A basic property of radial extension is that the index of the vector fields (or frames) constructed in this way is the same when measured in the strata or in the ambient space; this is called the Schwartz index of the vector field (or frame). MacPherson in [15] introduced the notion of local Euler obstruction, an invariant defined at each point of a singular variety using an index of an appropriate radial 1-form, and used this (among other things) to construct the homology Chern classes of singular varieties. Brasselet and Schwartz in [3] proved that the Alexander isomorphism $H^*(M, M \setminus V) \cong H_*(V)$ carries the Schwartz classes into the MacPherson classes; a key ingredient for this proof is their *proportionality theorem* relating the Schwartz index and the local Euler obstruction.

These were the first indices of vector fields and 1-forms on singular spaces, in the literature. Later in [8] was introduced another index for vector fields on isolated hypersurface singularities, and this definition was extended in [23] to vector fields on complete intersection germs. This is known as the GSV-index and one of its main properties is that it is invariant under perturbations of both, the vector field and the functions that define the singular variety. The definition of this index was recently extended in [4] for vector fields with isolated singularities on

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