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The moduli stack of rank-two Gieseker bundles with fixed determinant on a nodal curve

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§1. Introduction

Let $\{Y_t\}$ be a family of smooth curves degenerating to a nodal curve X_0. It is an interesting problem to consider how the moduli spaces of vector bundles on Y_t degenerate. Since the moduli space of vector bundles on the nodal curve X_0 is not compact, we need to find a good compactification. One way to compactify it is to add torsion-free sheaves. Another way, which is originally due to Gieseker [G] and developed by Nagaraj-Seshadri [NS] and Kausz [K2], is to add those vector bundles on a certain semistalbe model of X_0, which let us call Gieseker vector bundles. In these works they consider moduli spaces of vector bundles with fixed degree. In this paper we'd like to consider moduli spaces of vector bundles with fixed determinant.(See [Sun] for related results.)

This paper is heavily based on the work of Kausz [K1] [K2]. So, let me here explain his results briefly. In [K1], Kausz introduced a concept of generalized isomorphisms and showed that a projective variety KGl_n that is a compactification of Gl_n is the fine moduli space of generalized isomorphisms. Then in [K2] he showed that the normalization of the moduli space of Gieseker vector bundles on X_0 is a KGl_n-bundle over the moduli space of vector bundles on the normalization $\widetilde{X}_0$ of X_0. The purpose of this paper is to show that with the techniques invented by Kausz, we can also describe the structure of the moduli space of Gieseker vector bundles of rank 2 with fixed determinant on an irreducible nodal curve X_0.

The contents of the sections are as follows. In section 2 we introduce basic definitions. In section 3 we define θ-determinant generalized isomorphisms (only for rank 2 case), and see that the equivalence

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