

Amalgamated free product over Cartan subalgebra, II Supplementary Results & Examples

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§1 Introduction

Let $A \supseteq D \subseteq B$ be two von Neumann algebras together with a common Cartan subalgebra. Then the amalgamated free product $M = A *_D B$ with respect to the unique conditional expectations from A , B onto D can be considered. In our previous paper [U1], the questions of its factoriality and type classification were discussed in detail, which will be reviewed in §4. The main purpose of the paper is to give further supplementary results obtained after the completion of the previous paper together with discussions on some examples.

The author would like to express his sincere gratitude to the organizers Bruce Blackadar & Hideki Kosaki for inviting him to the US-Japan seminar 1999 held at Fukuoka, Japan and for giving opportunity to present this work.

§2 Amalgamated Free Products of von Neumann algebras

Let $A \supseteq D \subseteq B$ be σ -finite von Neumann algebras, and let $E_D^A : A \rightarrow D$, $E_D^B : B \rightarrow D$ be faithful normal conditional expectations. Then one can consider the amalgamated free product of A and B over D with respect to the conditional expectations E_D^A , E_D^B :

$$(M, E_D^M) = (A, E_D^A) *_D (B, E_D^B).$$

It is defined as a pair of a von Neumann algebra M into which the triple $A \supseteq D \subseteq B$ is embedded and a faithful normal conditional expectation $E_D^M : M \rightarrow D$, and characterized by the following three conditions:

- M is generated by the subalgebras A , B ;