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Some Remarks on the Infinitesimal Rigidity of the Complex Quadric

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Introduction

Let (X, g) be a compact Riemannian symmetric space. We say that a symmetric 2-form h on X satisfies the zero-energy condition if for all closed geodesics γ of X the integral

$$\int_\gamma h = \int_0^L h(\dot\gamma(s), \dot\gamma(s))ds$$

of h over γ vanishes, where $\dot\gamma(s)$ is the tangent vector to the geodesic γ parametrized by its arc-length and L is the length of γ. A Lie derivative of the metric g always satisfies the zero-energy condition. The space (X, g) said to be infinitesimally rigid if the only symmetric 2-forms on X satisfying the zero-energy condition are the Lie derivatives of the metric g.

Michel introduced the notion of infinitesimal rigidity in the context of the Blaschke conjecture, and proved that the real projective spaces $\mathbb{RP}^n$, with $n \geq 2$, and the flat tori of dimension ≥ 2 are infinitesimally rigid (see [17], [18] and [2]). Michel and Tsukamoto demonstrated the infinitesimal rigidity of the complex projective space $\mathbb{CP}^n$ of dimension $n \geq 2$ (see [17], [21], [6] and [7]); in fact, they proved that all the projective spaces which are not isometric to a sphere are infinitesimally rigid.

In [7] and [9], we showed that the complex quadric Q_n of dimension n is infinitesimally rigid when $n \geq 4$. In the monograph [12], we shall give a complete proof of the infinitesimal rigidity of the complex quadric Q_3 of dimension 3, which relies on the Guillemin rigidity of the Grassmannian of 2-planes in $\mathbb{R}^{n+2}$ proved in [10] and on results of Tela Nlenvo [20].

In this note, we present outlines of some new proofs of the infinitesimal rigidity of the complex quadric Q_n of dimension $n \geq 4$; the

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