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Appendix

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This is an appendix for the paper *"Infinitesimal logarithmic Torelli problem for degenerating hypersurfaces in $\mathbb{P}^n$"* by S. Saito. In Theorem (2-1), the injectivity of $d\rho_Z$ was proved for degenerating hypersurfaces. But $d\rho_Z$ is not injective in case n is odd and $\delta = 2$. We want to know the meaning of the exceptional cases. It is explained here, by using the extended period map, which is defined by K. Kato and S. Usui.

When we fix integers $\delta \geq 2$, $s \geq 1$ and $d \geq s\delta$, and general coefficients $a_\alpha \in \mathbb{C}$, a strict semistable degeneration $\tilde{X} \to B$ of hypersurfaces in $\mathbb{P}^{m+1}$ over the unit disk is constructed in Section 1. We denote the central fiber by $Z = Z_0 \cup Z_1 \cup \cdots \cup Z_s$, where $Z_0 = \tilde{X} \cap \tilde{H}_t$ and $Z_k = \tilde{X} \cap \mathbb{E}_k$.

Proposition 1. *Assume $d \geq s\delta + 1$. The mixed Hodge structure on $H^m(Z, \mathbb{Q})$ satisfies*

- $\mathrm{Gr}_l^W H^m(Z, \mathbb{Q}) = 0$ *if* $l \leq m - 2$,
- $\mathrm{Gr}_{m-1}^W H^m(Z, \mathbb{Q}) \simeq H^{m-1}_{\mathrm{prim}}(Z_0 \cap Z_s, \mathbb{Q})$.

$Z_0 \cap Z_s$ is a nonsingular hypersurface of degree δ in $\tilde{H}_t \cap \mathbb{E}_s \cong \mathbb{P}^m$.

Proof. The spectral sequence

$$E_1^{p,q} = H^q(Z^{[p]}, \mathbb{Q}) \Rightarrow H^{p+q}(Z, \mathbb{Q})$$

defines the weight filtration on $H^i(Z, \mathbb{Q})$, where $Z^{[p]} = \coprod_{0 \leq i_0 < \cdots < i_p \leq s} Z_{i_0} \cap \cdots \cap Z_{i_p}$. Let $\tilde{\mathbb{P}}_o = \tilde{H}_t \cup \mathbb{E}_1 \cup \cdots \cup \mathbb{E}_s$ be the central fiber of $\tilde{\mathbb{P}}_B \to B$. We know that $\tilde{\mathbb{P}}_o^{[p]} = Z^{[p]} = \emptyset$ for $p \geq 3$, so $\mathrm{Gr}_l^W H^m(Z, \mathbb{Q}) = 0$ for $l \leq m - 3$.

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