

A Proof of the Absolute Purity Conjecture (after Gabber)

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§0. Introduction

This article is an edited version of O. Gabber's talk on his proof of the absolute purity conjecture of A. Grothendieck given at the p -adic conference held in Toulouse in 1994. The details of the proofs given here are supplied by the author following marvelous ideas due to Gabber. The author takes the full responsibility for inaccuracies that may appear in this article.

The absolute purity conjecture is the following.

Conjecture (Grothendieck, [G]). *Let $Y \xrightarrow{i} X$ be a closed immersion of noetherian regular schemes of pure codimension c . Let n be an integer which is invertible on X , and let $\Lambda = \mathbf{Z}/n$. Then*

$$\mathcal{H}_Y^q(\Lambda) \simeq \begin{cases} 0 & \text{for } q \neq 2c, \\ \Lambda_Y(-c) & \text{for } q = 2c. \end{cases}$$

The conjecture has been proved in the following cases:

- a) X is smooth over a field k , and Y is also smooth over k ([AGV], exposé XVI, 3.7).
- b) X is of equal characteristic ([AGV], exposé XIX for special cases and conditional results, the general case can be deduced from Popescu's general Néron desingularization ([P]) as in §6).
- c) $\dim X \leq 2$ (Gabber (1976), see [Sa], §5, Remark 5.6 for a published proof).

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