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Some Geometric Methods in Commutative Algebra

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In this series of lectures, I will discuss three examples of techniques of algebraic geometry which have applications to commutative algebra. The examples chosen are those which pertain most closely to my own research interests.

§1. Hilbert functions, regularity and Uniform Artin-Rees

In this Section we describe results giving explicit, effective bounds for the Hilbert functions and postulation number of a Cohen-Macaulay module M of given dimension d and multiplicity e over a Noetherian local ring $(A, \mathfrak{m})$, with respect to a given $\mathfrak{m}$-primary ideal I. We also discuss related results bounding the Castelnuovo-Mumford regularity of the associated graded module of M with respect to I in terms of Hilbert coefficients, assuming only that M has positive depth; this leads to a new proof of the Uniform Artin-Rees theorem of Duncan and O'Carroll, and other results. The geometric technique used here is the cohomological study of the blow up of the ideal I, using in particular Grothendieck's formal function theorem.

1.1. The finiteness theorem for Hilbert functions

Recall that if $(A, \mathfrak{m})$ is a Noetherian local ring, M a finite A-module and $I \subset \mathfrak{m}$ an ideal of A such that M/IM has finite length, the *Hilbert function* (or more properly, the Hilbert-Samuel function) of M with respect to I is the numerical function

$$H_I(M)(n) = \ell(M/I^n M), \ \forall\, n \geq 0,$$

where we use the symbol ℓ to denote the length of a module (which has a finite composition series). Then there exists a corresponding *Hilbert polynomial*

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