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Either $71:35$ or $L_2(71)$ is a maximal subgroup of the Monster

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§1. Introduction

Let $\mathbb{M}$ be the Monster simple group. Then

$$|\mathbb{M}| = 2^{46}.3^{20}.5^{9}.7^{6}.11^{2}.13^{3}.17.19.23.29.31.41.47.59.71.$$

By [2] $71:35$ is the normalizer of a Sylow 71-subgroup and $59:29$ is the normalizer of a Sylow 59-subgroup of $\mathbb{M}$.

The purpose of this note is to prove:

Theorem 1. *Either* $71:35$ *or* $L_2(71)$ *is a maximal subgroup of* $\mathbb{M}$.

Theorem 2. *Either* $59:29$ *or* $L_2(59)$ *is a maximal subgroup of* $\mathbb{M}$.

Remark. $71:35$ is a maximal subgroup of $L_2(71)$ and $59:29$ is a maximal subgroup of $L_2(59)$. However we do not know whether $L_2(71)$ or $L_2(59)$ is involved in $\mathbb{M}$ or not (See [6]). Since $|L_2(71)| = 72.71.35$ and $|L_2(59)| = 60.59.29$, these are surprisingly small groups in comparison with $\mathbb{M}$.

Theorems 1 and 2 are closely related to the prime graphs of finite groups. Let G be a finite group and $\Gamma(G)$ the prime graph of G. $\Gamma(G)$ is the graph such that the vertex set is the set of prime divisors of $|G|$, and two distinct vertices p and r are joined by an edge if and only if there exists an element of order pr in G. Let $n(\Gamma(G))$ be the number of connected components of $\Gamma(G)$. It has been proved that $n(\Gamma(G)) \leq 6$ in [7], [4], [5].

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