Advanced Studies in Pure Mathematics 31, 2001
Taniguchi Conference on Mathematics Nara '98
pp. 93–125

Very Singular Diffusion Equations

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§1. Introduction

In the modeling of nonequilibrium phase transition it is often interesting to consider motion of phase-boundaries driven by singular surface energy. This topic was initiated by J. Taylor [T] and independently by S. Angenent and M. Gurtin [AG] who formulated motion of faceted curves moved by 'crystalline energy'. The governing equation is formally written in a quasilinear diffusion equation. However, because of singularity of energy, the diffusion effect is so strong that it may not be local. Even the notion of solution is not clear in general. There are two ways to handle such very singular diffusion equations systematically as a limit of diffusion equation with smooth energy. The first one is variational approach or the theory of nonlinear semigroups initiated by Y. Kōmura [Ko] and developed by many mathematicians for many years. It provides mathematical formulation of various important problems including the Stefan problem and the Hele-Shaw problem as explained in a book of A. Visintin [V]. The application of this theory to motion with facets is found in [FG] and is further developed by [EGS]; the theory developed in [HZ] is in the line of this approach. The second one is an approach by extending the theory of viscosity solutions initiated by M.-H. Giga and Y. Giga [GG1], [GG3], [GG4]. The first method applies to problem for arbitrary dimensions but the method needs the divergence structure of equations. The second method is so far limited in one space dimension and spatially homogeneous problem. However, it does not require divergence structure of the equation so that it applies equations

Received May 27, 1999.
[1] Partly supported by Nissan Science Foundation and the Grant-in-Aid for Science Research No. 10304010, No. 08874005, the Japan Ministry of Education, Science and Culture.
[2] Partly supported by the Grant-in-Aid for Science Research No. 09650074, the Japan Ministry of Education, Science and Culture.