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Finiteness of a certain Motivic Cohomology Group of Varieties over Local and Global Fields

Kanetomo Sato

INTRODUCTION

In this paper, I would like to survey my recent research [22]. I would like to express gratitude to the organizers for giving me this opportunity to write this manuscript.

Let k be a global field, i.e., an algebraic number field (case (N)) or a function field in one variable over a finite field (case (F)). Let X be a projective smooth geometrically connected k-variety. Let l be a prime number invertible in k. The l-adic regulator map of Soulé [24]

$$r_l^{i,n} : \mathrm{H}^i_{\mathcal{M}}(X, \mathbb{Q}(n))_{\mathbb{Q}_l} \to \mathrm{H}^i_{\mathrm{cont}}(X, \mathbb{Q}_l(n)).$$

is a central topic in the arithmetic geometry. Here $\mathrm{H}^i_{\mathcal{M}}(X, \mathbb{Q}(n))$ denotes the motivic cohomology and is defined by the n-th Adams eigenspace of the algebraic K-group $K_{2n-i}(X)_{\mathbb{Q}}$ ([17] and [25]), and the right hand side is the continuous etale cohomology group (cf. Jannsen [9]). The coefficient $\mathbb{Q}_l(n)$ in the right hand side means the n-th Tate twist of $\mathbb{Q}_l$. In the case $i = 2n$, it is known that this map coincides with the cycle map for the Chow group of algebraic cycles of codimension n modulo rational equivalence ([9] 6.14):

$$\mathrm{cl}_l : \mathrm{CH}^n(X)_{\mathbb{Q}_l} \to \mathrm{H}^{2n}_{\mathrm{cont}}(X, \mathbb{Q}_l(n)).$$

We write $\mathrm{F}^\bullet$ for the Hochschild–Serre filtration on the continuous etale cohomology group w.r.t. the covering $X \otimes_k k^{\mathrm{sep}} \to X$. For instance, F^2 of $\mathrm{H}^i_{\mathrm{cont}}(X, \mathbb{Q}_l(n))$ is defined by the image of the Hochschild–Serre mapping

$$\mathrm{H}^2_{\mathrm{cont}}(G_k, \mathrm{H}^{i-2}_{\mathrm{et}}(\overline{X}, \mathbb{Q}_l(n))) \to \mathrm{H}^i_{\mathrm{cont}}(X, \mathbb{Q}_l(n)),$$

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