

A Survey of p -Extensions

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This is a brief survey of what is known or unknown about the Galois group of the maximal pro- p -extension (p a fixed prime) of a number field which is unramified outside a given set of places. We are particularly interested in

- presentation in terms of generators and relations
- cohomological dimension

of the Galois group. The contents are as follows. In Section 1 we recall basic facts on pro- p -groups. In Section 2 we review the structure of the Galois group of the maximal pro- p -extension of a local field. In Section 3 we state some known facts and unsolved conjectures about the structure of the Galois group of the the maximal pro- p -extension of a number field which is unramified outside a given finite set of places. In Section 4 we introduce some topics in Iwasawa theory. In Section 5 we state some known facts about the structure of the Galois group of the maximal pro- p -extension of a number field. Finally, as an application of Sections 3 and 4, we give some examples of free pro- p -extensions of number fields in Section 6.

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§1. Pro- p -groups

Main references are Serre [54, I §3–§4] and Koch [26, §5–§6]. Let G be a pro- p -group.

1.1. Generators and relations

We put $d(G) = \dim H^1(G, \mathbb{Z}/p\mathbb{Z})$ and $r(G) = \dim H^2(G, \mathbb{Z}/p\mathbb{Z})$. $d(G)$ is the minimal number of generators of G , which we also call the rank of G , and $r(G)$ is the minimal number of relations of G .

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