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# On Artin $L$-Functions

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## §1. Introduction.

The celebrated conjecture of Emil Artin about the holomorphy of his non-abelian $L$-series has inspired a vast amount of development in number theory, algebraic geometry and representation theory. There exist at least four different programs to approach this conjecture. Most notable amongst these is the famous Langlands program. One of the objectives of the Langlands program has been to create the theoretical framework with which to attack Artin's conjecture. The notions of base change, automorphic induction and converse theory provide the conceptual tools to be developed and applied towards this goal. There are already excellent descriptions of this approach in the literature, such as Gelbart [Ge], Murty [Mu1], Prasad and Yogananda [PY] and the recent paper by Rogawski [Rog]. Therefore, we shall not deal with this approach in this survey.

A second method is the program initiated by Serre [Se2]. Indeed, Khare [Kh] has recently shown that Serre's conjectures imply Artin's conjecture for two-dimensional, complex, odd representations over $\mathbb{Q}$. After the spectacular success of Wiles, Buzzard and Taylor [BT] proved a theorem that makes a significant advance towards the $A_5$ case of Artin's conjecture. We will refer the reader to these papers as well as [ST] for an insight into these new $p$-adic methods. In this paper, we shall focus more on the analytic aspects of Artin $L$-series and describe the approach implied by the recently formulated conjectures of Selberg concerning general $L$-functions with Euler products and functional equations. At the end of the paper, we discuss certain group-theoretic considerations

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