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A Duality of a Twisted Group Algebra of the Hyperoctahedral Group and the Queer Lie Superalgebra

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§1. Introduction

We establish a duality relation (Theorem 4.2) between one of the twisted group algebras of the hyperoctahedral group H_k (or the Weyl group of type B_k) and a Lie superalgebra $\mathfrak{q}(n_0) \oplus \mathfrak{q}(n_1)$ for any integers $k \geq 4$ and n_0, $n_1 \geq 1$. Here $\mathfrak{q}(n_0)$ and $\mathfrak{q}(n_1)$ denote the "queer" Lie superalgebras as called by some authors. The twisted group algebra $\mathcal{B}'_k$ in focus in this paper belongs to a different cocycle from the one $\mathcal{B}_k$ used by A. N. Sergeev in his work [8] on a duality with $\mathfrak{q}(n)$ and by the present author in a previous work [11]. This $\mathcal{B}'_k$ contains the twisted group algebra $\mathcal{A}_k$ of the symmetric group $\mathfrak{S}_k$ in a straightforward manner (cf. §1. 1. 1), and has a structure similar to the semidirect product of $\mathcal{A}_k$ and $\mathbb{C}[(\mathbb{Z}/2\mathbb{Z})^k]$. ($\mathcal{B}'_k$ and $\mathcal{B}_k$ were denoted by $\mathbb{C}^{[-1,+1,+1]}W_k$ and $\mathbb{C}^{[+1,+1,-1]}W_k$ respectively by J. R. Stembridge in [10].)

In §2, we construct the $\mathbb{Z}_2$-graded simple $\mathcal{B}'_k$-modules (where $\mathbb{Z}_2 = \mathbb{Z}/2\mathbb{Z}$) using an analogue of the little group method. These simple $\mathcal{B}'_k$-modules are slightly different from the non-graded simple $\mathcal{B}'_k$-modules constructed by Stembridge in [10] because of the difference between $\mathbb{Z}_2$-graded and non-graded theories, but they can easily be translated into each other. We will use the algebra $\mathcal{C}_k \dot{\otimes} \mathcal{B}'_k$, where $\mathcal{C}_k$ is the 2^k-dimensional Clifford algebra (cf. (3.2)) and $\dot{\otimes}$ denotes the $\mathbb{Z}_2$-graded tensor product (cf. [1], [2], [11, §1]), as an intermediary for establishing our duality, as we explain below. The construction of the simple $\mathcal{B}'_k$-modules leads to a construction of the simple $\mathcal{C}_k \dot{\otimes} \mathcal{B}'_k$-modules in §3.

In §4, we define a representation of $\mathcal{C}_k \dot{\otimes} \mathcal{B}'_k$ in the k-fold tensor product $W = V^{\otimes k}$ of $V = \mathbb{C}^{n_0+n_1} \oplus \mathbb{C}^{n_0+n_1}$, the space of the natural representation of the Lie superalgebra $\mathfrak{q}(n_0 + n_1)$. This representation

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