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Deformation Theory of CR-Structures and Its Application to Deformations of Isolated Singularities II

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Introduction

Deformations of an analytic variety with only isolated singular points induce deformations of strongly pseudo-convex CR structures on its link. It is M. Kuranishi who initiated to consider deformations of compact strongly pseudo-convex CR structures expecting to describe deformations of isolated singular points of analytic varieties. Since nonequivalent CR manifolds can bound the same isolated singular point, we consider deformations of CR structures up to equivalence weaker than the CR-equivalence. This equivalence is induced from wiggling in a complex manifold and we will call the deformation theory of CR structures under that equivalence the Kuranishi deformation theory of CR structures. In [Ku3], [Ku4], M. Kuranishi obtained a C^{∞} -family of deformations of the CR structure on a compact strongly pseudo-convex CR manifold of real dimension five or higher, continuing his early works on deformations of compact complex structures ([Ku1], [Ku2]). We consider holomorphic families of CR structures. In the first half of this survey, we will review the holomorphically parametrized deformation theory of strongly pseudo-convex CR structures developped by T. Akahori et al. ([Ak1], [Ak2], [Ak3], [Ak4], [Ak-My1], [Ak-My2], [Ak-My3], [Ak-My4], [Bu-Ml], [My1], [My2], [My3]) and its relationship with algebraic deformation theory of isolated singularities ([Do], [Gr], [Tj]).

The relationship between compact strongly pseudo-convex CR manifolds and isolated singularities is based on the fact that an embeddable compact strongly pseudo-convex CR manifold bounds a unique normal Stein complex space ([Ha-La]) and all compact strongly pseudo-convex

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