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Some Remarks on Compact Strongly Pseudoconvex CR Manifolds

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Dedicated to Professor M. Kuranishi on his 70th birthday

§0. Introduction.

In this note, we make some remarks on compact strongly pseudoconvex CR manifolds. These remarks are related to the problem of minimal embedding dimension of a compact strongly pseudoconvex CR manifold in complex Euclidean space and the classification problem of compact strongly pseudoconvex CR manifolds. Most are contained in [LuY1-3] and in our joint paper with Yung Yu [LuYY]. We hope that this expository note would be of interest for the study of the relationship between the geometry of a compact strongly pseudoconvex CR manifold and the singularities that it may bound, much in the spirit of Kuranishi's application of $\overline{\partial}_b$ to the deformation of isolated singularities [Ku1]. The first author most gratefully recalls the years at Columbia when he studied with Professor Kuranishi and was brought into the fascinating field of CR geometry, being inspired by Professor Kuranishi's mathematical power and depth.

§1 Preliminary

Definition 1.1. *Let X be a connected real manifold of dimension $2n-1$, $n \geq 2$. A CR structure on X is an $(n-1)$-dimensional subbundle S of the complexified tangent bundle $\mathbb{C}TX$ such that*

(a) $S \cap \overline{S} = \{0\}$,

(b) *If L, L' are local sections of S, then so is $[L, L']$.*

Definition 1.2. *Let $L_1, \ldots, L_{n-1}$ be a local frame of S. Then $\overline{L}_1, \ldots, \overline{L}_{n-1}$ is a local frame of $\overline{S}$ and one may choose a local section N*

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