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Absolute Continuity of the Essential Spectrum for some Linearized MHD Operator

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§1. Introduction

The magnetohydrodynamic (MHD) motion of plasma is described by the system of equations which consist of the compressible Euler equation and the reduced Maxwell equation with the mutual interaction terms given by the Lorenz force and Ohm's low. Related to the plasma confinement experiment, the study of the behavior of plasma motion around the equilibrium is very important. The MHD motion in the vicinity of the equilibrium is described by the following linearized MHD equation:

$$
\begin{aligned}
(1.1)\qquad \rho_0 \frac{\partial^2 \xi}{\partial t^2} &= -K\xi \equiv \operatorname{grad}\{\gamma P_0 (\operatorname{div} \xi) + (\operatorname{grad} P_0)\cdot \xi\} \\
&\quad + B_0 \times \operatorname{rot}(\operatorname{rot}(B_0 \times \xi)) - (\operatorname{rot} B_0) \times \operatorname{rot}(B_0 \times \xi),
\end{aligned}
$$

for the Lagrangian displacement vector field $\xi : \Omega \subset R^3 \to R^3$. Here, the equilibrium quantities ρ_0 (=density), P_0 (=pressure), B_0 (=magnetic field), are given bounded smooth functions which satisfy the equilibrium condition:

$$
(1.2)\qquad
\begin{aligned}
&\operatorname{grad} P_0 = j_0 \times B_0, \quad \operatorname{div} B_0 = 0, \\
&\text{with } j_0 = \operatorname{rot} B_0 \text{ (=electric current density)}, \\
&P_0 \geq c_P > 0, \quad \rho_0 \geq c_\rho > 0 : \text{arbitrary}.
\end{aligned}
$$

We assume in (1.1) that the specific heat ratio γ is a positive constant. We impose a slip condition: $\xi \cdot n = 0$ on the boundary $\partial\Omega$ where n is the unit normal on the boundary.

In this paper, we shall study some spectral properties of the operator $\rho_0^{-1} K$ in a Hilbert space $L^2(\Omega; \rho_0 dr)^3$. In particular, we shall prove the absolute continuity of the essential spectrum and the discreteness of the embedded eigenvalues in the continuum under some assumptions on the

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