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On Scattering by Two Degenerate Convex Bodies

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§1. Introduction

Let n be an odd integer ≥ 3, and let $\mathcal{O}$ be a bounded open set in $\mathbb{R}^n$ such that

$$\Omega = \mathbb{R}^n - \overline{\mathcal{O}} \quad \text{is connected.} \tag{1.1}$$

We assume that

$$\Gamma = \partial\mathcal{O} \quad \text{is smooth.}$$

Denote by $\mathcal{S}(z)$ the scattering matrix for $\mathcal{O}$. The scattering matrix $\mathcal{S}(z)$ is an $\mathcal{L}(L^2(S^{n-1}))$-valued holomorphic function defined in $\{z \in \mathbb{C};\ \mathrm{Re}\, z < 0\}$, where we denote by $\mathcal{L}(E)$ the space of all the bounded operators from E into itself. As a fundamental property of the scattering matrix, it is shown in Lax-Phillips [7]:

Theorem 5.1 of Chapter V. *The scattering matrix $\mathcal{S}(z)$ is holomorphic on the real axis and meromorphic in the whole plane, having a pole at exactly those points z for which there is a nontrivial z-outgoing local solution of*

$$\begin{cases} (\triangle + z^2)\, u = 0 & \text{in } \Omega \\ \quad u = 0 & \text{on } \Gamma. \end{cases}$$

In the study of scattering by obstacles, the problem to know relationships between the geometry of obstacles and the distribution of poles of scattering matrices is one of the most interesting and important problems. It is conjectured that the more rays of geometric optics are trapped by $\mathcal{O}$ the more solutions of the wave equation are trapped by $\mathcal{O}$, and that the more solutions of the wave equation are trapped, the nearer to the real axis it appears the poles of the scattering matrix.

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