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Scattering Theory in the Energy Space for a Class of Nonlinear Wave Equations

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Dedicated to Professor ShigeToshi Kuroda on his sixtieth birthday

§1. Introduction

The purpose of this talk is to present a survey of the theory of scattering for a class of nonlinear wave equations of the form

$$\square\varphi \equiv \partial_t^2\varphi - \Delta\varphi = -f(\varphi) \tag{1.1}$$

in a space of initial data and asymptotic states as large as the energy space associated with that equation. The exposition will follow the treatment given in [12]. Here φ is a complex valued function defined in space time $\mathbb{R}^{n+1}$, Δ is the Laplace operator in $\mathbb{R}^n$, and f is a nonlinear suitably regular complex valued function satisfying polynomial bounds at zero and at infinity. A large amount of work has been devoted to the theory of scattering for the equation (1.1) and for several other equations, and we shall devote most of this introduction to a partial review of nonlinear scattering in order to put the subsequent treatment of (1.1) into perspective.

The general setting is the following. One considers a semilinear equation

$$\partial_t u = Lu + F(u) \tag{1.2}$$

where L is a linear antiselfadjoint operator in some Hilbert space $\mathcal{H}$, and generates a one parameter unitary group $U(t) = \exp(tL)$ in $\mathcal{H}$. One is interested in situations where the global Cauchy problem for (1.2) is well understood in some space X (which may or may not coincide with $\mathcal{H}$). In particular any initial data $u_0 \in X$ should generate a unique global X valued solution of (1.2) with $u(0) = u_0$ and with suitable regularity

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