Advanced Studies in Pure Mathematics 22, 1993
Progress in Differential Geometry
pp. 133–152

Non-Commutative Complex Projective Space

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§0. Introduction

The concept of quantized manifolds has much interest from a geometrical point of view. In fact, quantum groups [6] and non-commutative tori [4] [12] are typical examples in this spirit. One approach to constructing quantized manifolds is based on the deformation quantization introduced by Bayen et al [1]. This is the deformation of the Poisson algebra of functions on a symplectic manifold via a star product.

However, deformation quantization providing only an algebraic description does not seem to describe the "underlying space" adequately. From the geometric point of view, we want to construct something like non-commutative manifolds which just represent the quantum state space.

For this purpose, we introduced the notion of Weyl manifolds [10], [11] as a prototype of non-commutative manifolds. A Weyl manifold W_M is defined as a certain algebra bundle over a symplectic manifold M with the formal Weyl algebra as the fiber. The star product given by the deformation quantization is realized on a certain class of sections on W_M, called *Weyl functions.* We present in this paper a *non-commutative complex projective space* $W_{P_n(\mathbf{C})}$ as an example of a Weyl manifold.

There are two ways of constructing star products on $P_n(\mathbf{C})$. The first is *intrinsic*, and was initiated by Berezin [2], who gave a covariant symbol calculus for certain operators acting on local holomorphic functions on the 2-sphere and on the Lobachevskii plane, and defined the star product on these spaces by using the symbol calculus. Moreno [9] and Cahen-Gutt-Rawnsley [3] extended these ideas to Kaehler symmetric spaces.

Received February 15, 1991.
Revised May 27, 1991.