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Letter to J. Dieudonne

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Professor J. Dieudonne
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Dear Professor Dieudonne:

A few days ago, I received a letter from Professor A. Weil, asking me to send you a copy of a letter I wrote him the other day and to give you a brief account of my result on L-functions. I, therefore, enclose here a copy of that letter and write an outline of my idea on L-functions.

Let k be a finite algebraic number field, J the idele group of k, topologized as in a recent paper of Weil. J is a locally compact abelian group containing the principal idele group P as a discrete subgroup. We denote by J_0 the subgroup of J consisting of ideles $\mathfrak{a} = (a_p)$ such that $a_p = 1$ for all infinite (i.e. archimedean) primes P. We call J_0 the finite part of J and define the infinite part J_∞ similarly, so that we have

$$J = J_0 \times J_\infty, \quad \mathfrak{a} = \mathfrak{a}_0\mathfrak{a}_\infty, \quad \mathfrak{a}_0 \in J_0, \quad \mathfrak{a}_\infty \in J_\infty.$$

We also denote by U the compact subgroup of J consisting of ideles $\mathfrak{a} = (a_p)$ such that the absolute value $|a_p|_p = 1$ for every prime P. $U_0 = U \cap J_0$ is then an open, compact subgroup of J_0 and J_0/U_0 is canonically isomorphic to the ideal group I of k. According to Artin-Whaples, we can choose the absolute values $|a_p|_p$ so that the volume function $V(\mathfrak{a}) = \Pi_p |a_p|_p$ $(\mathfrak{a} = (a_p))$ has the value 1 at every principal idele $\alpha \in P$ (the product formula) and that $V(\mathfrak{a}_0)^{-1}$ is equal to the absolute norm $N(\tilde{\mathfrak{a}}_0)$ of the ideal $\tilde{\mathfrak{a}}_0$, which corresponds to $\mathfrak{a}_0$ by the above isomorphism between J_0/U_0 and I.