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Billiards without Boundary and Their Zeta Functions

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In this article we consider a simple dynamical system in R^2 with elastic reflection. As it is shown in [2], the zeta function

$$\zeta(s) = \prod_{\gamma}(1 - \exp[-sT_\gamma])^{-1} \tag{1}$$

of such a dynamical system satisfies nice properties which enable us to apply the results in [3] and [4], where $\prod_\gamma$ is taken over all prime periodic orbits of the dynamical system and T_γ denotes their period. Our present purpose is to summarize auxiliary results which do not appear in [2].

First we recall definitions and notations in [2]. Let $O_1, O_2, ..., O_L$ ($L \geq 3$) a finite number of bounded domains in R^2, which will be called scatterers, with smooth boundary. We impose the following hypotheses on scatterers:

(H.1) (dispersing) For each j, O_j is strictly convex, i.e., the boundary ∂O_j is a simple closed curve with nonvanishing curvature.

(H.2) (no eclipse) For any triple of distinct indices (j, k, l), the convex hull of $\overline{O_j}$ and $\overline{O_k}$ does not intersect $\overline{O_l}$.

Under these hypotheses it is clear that the boundary ∂O of $O = \cup_{j=1}^{L} O_j$ equals $\cup_{j=1}^{L} \partial O_j$.

Let $SR^2 = R^2 \times S^1 = \{(q, v)\,; |v| = 1\}$ be the unit tangent bundle of R^2, and let $\pi : SR^2 \to R^2$ be the natural projection. Choose a point $q_j \in \partial O_j$ and fix it for each j. We define the following quantities for $x = (q, v) \in \partial O$.

$$\begin{aligned} &\xi_0(x) = \xi_0(q) = j \ \text{ if } \pi(x) \in \partial O_j, \text{ and} \\ &r(x) = r(q),\ \phi(x) = \phi(q, v),\ k(x) = k(q), \end{aligned} \tag{2}$$

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