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Solvable Lattice Models and Algebras of Face Operators

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§1. Introduction

In this paper, we discuss algebras associated with the solvable lattice models and conformal field theory (CFT). In [7], solutions of Yang-Baxter equations (YBE) associated with the vector representations of simple Lie algebras are introduced. For each solution, we denote by Y_n an algebra of Yang-Baxter operators. On the other hand, solutions of star-triangle relations (or IRF models) associated with the vector representations of classical simple Lie algebras are introduced in [9]. For each solution, an algebra of face operators F_n are introduced in Section 3. Let Y_n and F_n be the above algebras associated with a simple Lie algebra of type A. Then they are both quotients of Iwahori's Hecke algebra $H_n(q)$ of type A. Let $\mathfrak{g}$ be a simple Lie algebra of type B, C or D. For the algebras Y_n and F_n associated with $\mathfrak{g}$, we show in Section 5 that Y_n and F_n are both quotients of a q-analogue of Brauer's centralizer algebra $C_n(a,q)$. The algebras $H_n(q)$ and $C_n(a,q)$ have the following properties.

(1) The algebra of face operators of an IRF model associated with a classical simple Lie algebra is a quotient of $H_n(q)$ or $C_n(a,q)$.
(2) The algebra of Yang-Baxter operators associated with a classical simple Lie algebra is a quotient of $H_n(q)$ or $C_n(a,q)$. ([7], [13])
(3) Let $U_q(\mathfrak{g})$ denote the q-analogue of the universal enveloping algebra of a simple Lie algebra $\mathfrak{g}$. Then the centralizer algebra associated with the vector representation of $U_q(\mathfrak{g})$ is equal to a quotient of $H_n(q)$ if $\mathfrak{g}$ is of type A and a quotient of $C_n(a,q)$ if $\mathfrak{g}$ is of type B or C. The centralizer algebra associated with that of type D contains a quotient of $C_n(a,q)$. (Immediate consequence of (2).)

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