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Einstein-Kähler Metrics on Minimal Varieties of General Type and an Inequality between Chern Numbers

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In this paper, we shall give a complete proof of the results which were announced in Part I in this volume. For earlier results on Einstein-Kähler metrics and tangent sheaves of minimal varieties, see the introduction of [Sg].

The contents of this paper are as follows. In Section 1 through 4, solving a degenerate Monge-Ampère equation, we shall construct a family of Einstein-Kähler metrics on the smooth part of minimal varieties of general type. In Section 5, we shall show a subsequence of this family of Einstein-Kähler metrics converges to an Einstein-Kähler metric, whose cohomology class corresponds properly to a negative constant multiple of the first Chern class of the variety. In Section 6, an inequality between Chern numbers for minimal varieties, so called Bogomolov-Gieseker type inequality, will be proved. In Sections 7 and 8, using the metric constructed in Section 5, we shall obtain a sufficient condition for the tangent sheave of certain varieties to be stable.

After the completion of this work, we were informed that S. Bando and R. Kobayashi obtained, simultaniously with ours, the same result as Theorem 5.6 by a heat equation method which is different from ours. We added Theorem 5.8 according to their suggestions via correspondences. I wish to express my hearty gratitude to them.

§1. A degenerate Monge-Ampère equation

Let M be an n dimensional compact projective algebraic manifold and

$$E = \{E_i\}_{i=1}^N$$

effective divisors on M. Assume that the following condition is satisfied.

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