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A Note on Norm-Coherent Units in Certain Z_p-Extensions

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Dedicated to K. Iwasawa, on the occasion of his seventieth birthday

Let K be a number field and L a finite abelian extension of K. One of the major open problems in number theory is to construct a large group of units in L, using transcendental functions associated with K. This has only been accomplished when K is Q or imaginary quadratic, two cases that are distinguished by the absence of units in the base other than roots of unity. Indeed, if we seek a procedure suitable for a wide variety of K, it should rather *not* produce units already in the base field. We therefore introduce the group E_L^0 of units in L whose norm to K is 1, and ask for an explicit group C_L of finite index in E_L^0. In the two mentioned cases C_L is the group of circular or elliptic units, and $[E_L^0: C_L]$ is equal to the class number $h(L)$ (in the elliptic case) or $h(L^+)$ (wnen $K=Q$ and L^+ is the real subfield of L), times a well understood "fudge factor" (see [3] and [8]).

Another feature of both circular and elliptic units is that they appear in norm-coherent sequences in Z_p extensions. This is significant for two reasons. First, we can base a proof of the fact that they are units on their norm-compatibility. Secondly, for *any* norm coherent sequence of units, R. Coleman associates in [2] a certain "generating power series" with coefficients in the completion of K. If the sequence happens to come from (C_{L_n}) ((L_n) is the Z_p-tower), this power series is the Taylor expansion of the global function giving the special units. Thus the Coleman power series is an infinitesimal object, midway between the individual unit and the global, transcendental, generating function.

In this note we study units in certain Z_p extensions obtained from torsion points on abelian varieties with complex multiplication. We prove that there are "many" norm coherent sequences of units. In view of the remarks made above, this may be seen as (weak) evidence for the existence of "abelian units".

I would like to thank R. Coleman for suggesting the problem to me.

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